Lecture 3: The Electric Field

The Electric Charge Density

We should distinguish three types of the charge density.

1. The ratio of the charge value to volume is called the volume charge density:

$$\rho = \frac{dq}{dV}, [\rho] = \frac{C}{\mathrm{m}^3}. \quad (1.4)$$

2. The ratio of the charge value to area, over which it is distributed, is called the surface charge density:

$$\sigma = \frac{dq}{dS}, [\sigma] = \frac{C}{\mathrm{m}^2}. \quad (1.5)$$

3. The ratio of the charge value to line, along which it is distributed, is called the line charge density:

$$\lambda = \frac{dq}{dl}, [\lambda] = \frac{C}{\mathrm{m}}. \quad (1.6)$$

The Electric Field

The temperature at every point in a room has a definite value.You can measure the temperature at any given point or combination of points by putting a thermometer there.We call the resulting distribution of temperatures a *temperature field.*

The electric field is a *vector field;* it consists of a distribution of *vectors,* one for each point in the region around a charged object, such as a charged rod. In principle, we can define the electric field at some point near the charged object, such as point *P* in Fig. 22-1*a*, as follows: We first place a *positive* charge *q*0, called a *test charge,* at the point.We then measure the electrostatic force that acts on the test charge. Finally, we define the electric field at point *P* due to the charged object as

$$\vec{E} = \frac{\vec{F}}{q_0}, \quad (1.7)$$

where $q_0 = +1$ C. The direction of $\vec{E}$ vector coincides with that of the force acting on the positive charge in the electric field. The unit of electric field intensity in **SI** system is

$$[E] = \frac{\text{N}}{\text{C}} = \frac{\text{V}}{\text{m}}. \qquad (1.8)$$

Hence, it is possible to describe an electric field by its ***intensity E***, which is defined as follows

$$\vec{E} = \frac{q}{4\pi\varepsilon_o r^2} \cdot \frac{\vec{r}}{|\vec{r}|} \quad . \qquad (1.9)$$

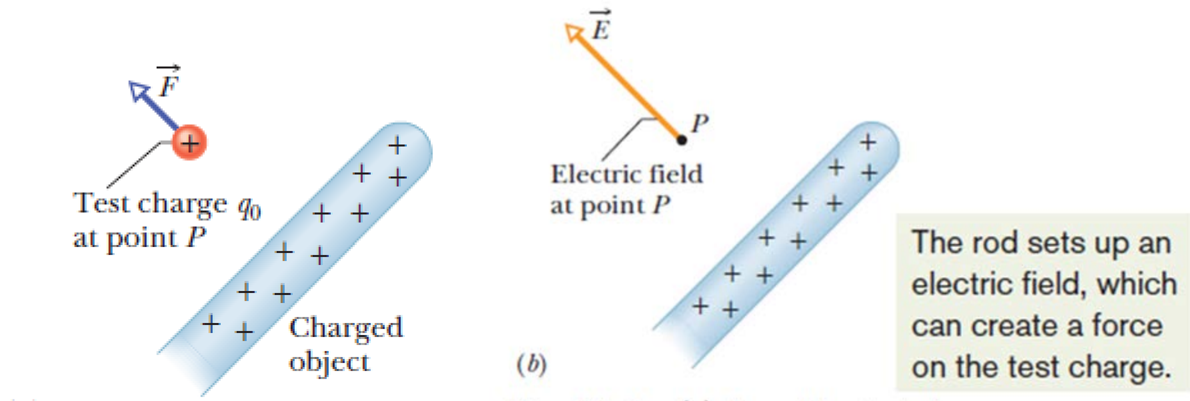


Fig. 22-1 (*a*) A positive test charge

The electric field from a positive charge points away from the charge; the electric field from a negative charge points toward the charge. Like the electric force, the electric field ***E*** is a vector. If the electric field at a particular point is known, the force a charge ***q*** experiences when it is placed at that point is given by

$$\vec{F} = q \cdot \vec{E} \quad . \qquad (1.10)$$

If ***q*** is positive, the force is in the same direction as the field; if ***q*** is negative, the force is in the opposite direction as the field.

The electric field of fixed charges is called ***the electrostatic field***. We should distinguish the homogeneous and heterogeneous fields. At any point of homogeneous field the vector $\vec{E}$ is the same. On the contrary, at any point of heterogeneous field there are different $\vec{E}$-vectors. The ***superposition principle*** takes place: the vector $\vec{E}$ at a given point for electric field of any system of charges may be found by summing the vectors for the individual charges

$$\vec{E} = \sum_{i=1}^{n} \vec{E}_i \quad , \qquad (1.11)$$

where ***n*** is the number of charges, fig 1.9.

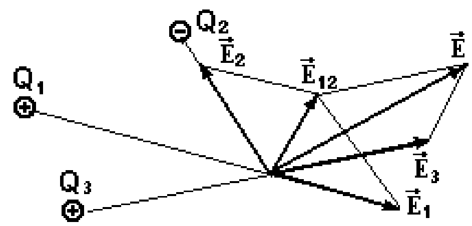


Figure 1.9

Electric Field of a Point Charge

If the source distribution is a point charge q, it is easy to find the electric field that it produces. We call the location of the charge the **source point,** and we call the point P where we are determining the field the **field point.** It is also useful to introduce a *unit vector* $\hat{r}$ that points along the line from source point to field point (Fig. 21.17a). This unit vector is equal to the displacement vector $\vec{r}$ from the source point to the field point, divided by the distance $r = |\vec{r}|$ between these two points; that is, $\hat{r} = \vec{r}/r$. If we place a small test charge q_0 at the field point P, at a

distance r from the source point, the magnitude F_0 of the force is given by Coulomb's law, Eq. (21.2):

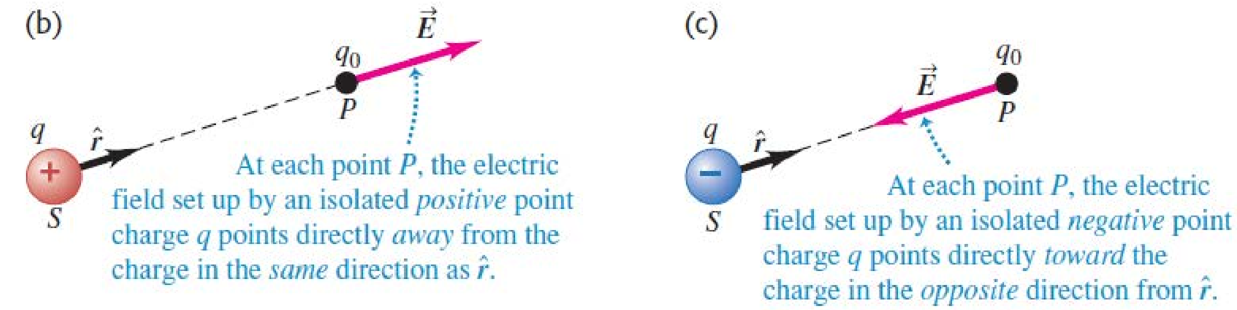
$$F_0 = \frac{1}{4\pi\epsilon_0}\frac{|qq_0|}{r^2}$$

From Eq. (21.3) the magnitude E of the electric field at P is

$$E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} \qquad \text{(magnitude of electric field of a point charge)} \qquad (21.6)$$

Using the unit vector $\hat{r}$, we can write a *vector* equation that gives both the magnitude and direction of the electric field $\vec{E}$:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \qquad \text{(electric field of a point charge)} \qquad (21.7)$$



The electric field $\vec{E}$ produced at point P by an isolated point charge q at S. Note that in both (b) and (c), $\vec{E}$ is *produced* by q

A more complete understanding of the electric field concept can be gained by considering the field created by a point charge, as in the following example.

Example 10 The Electric Field of a Point Charge

There is an isolated point charge of $q = +15\ \mu C$ in a vacuum at the left in Figure 18.19*a*. Using a test charge of $q_0 = +0.80\ \mu C$, determine the electric field at point P, which is 0.20 m away.

Reasoning Following the definition of the electric field, we place the test charge q_0 at point P, determine the force acting on the test charge, and then divide the force by the test charge.

Solution Coulomb's law (Equation 18.1), gives the magnitude of the force:

$$F = k\frac{|q_0||q|}{r^2} = \frac{(8.99 \times 10^9\ N \cdot m^2/C^2)(0.80 \times 10^{-6}\ C)(15 \times 10^{-6}\ C)}{(0.20\ m)^2} = 2.7\ N$$

Equation 18.2 gives the magnitude of the electric field:

$$E = \frac{F}{|q_0|} = \frac{2.7\ N}{0.80 \times 10^{-6}\ C} = \boxed{3.4 \times 10^6\ N/C}$$

The electric field $\vec{E}$ points in the *same direction* as the force $\vec{F}$ on the positive test charge. Since the test charge experiences a force of repulsion directed to the right, the electric field vector also points to the right, as Figure 18.19*b* shows.

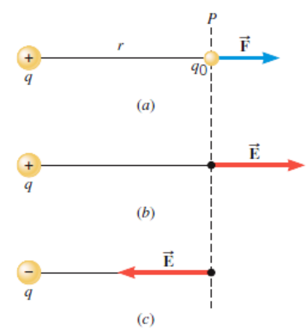


Figure 18.19 (*a*) At location *P*, a positive test charge q_0 experiences a repulsive force $\vec{F}$ due to the positive point charge *q*. (*b*) At *P*, the electric field $\vec{E}$ is directed to the right. (*c*) If the charge *q* were negative rather than positive, the electric field would have the same magnitude as in (*b*) but would point to the left.

Example 21.8 Field of an electric dipole

Point charges $q_1 = +12$ nC and $q_2 = -12$ nC are 0.100 m apart (Fig. 21.22). (Such pairs of point charges with equal magnitude and opposite sign are called *electric dipoles*.) Compute the electric field caused by q_1, the field caused by q_2, and the total field (a) at point *a*; (b) at point *b*; and (c) at point *c*.

SOLUTION

IDENTIFY and SET UP: We must find the total electric field at various points due to two point charges. We use the principle of superposition: $\vec{E} = \vec{E}_1 + \vec{E}_2$. Figure 21.22 shows the coordinate system and the locations of the field points *a*, *b*, and *c*.

EXECUTE: At each field point, $\vec{E}$ depends on $\vec{E}_1$ and $\vec{E}_2$ there; we first calculate the magnitudes E_1 and E_2 at each field point. At *a* the magnitude of the field $\vec{E}_{1a}$ caused by q_1 is

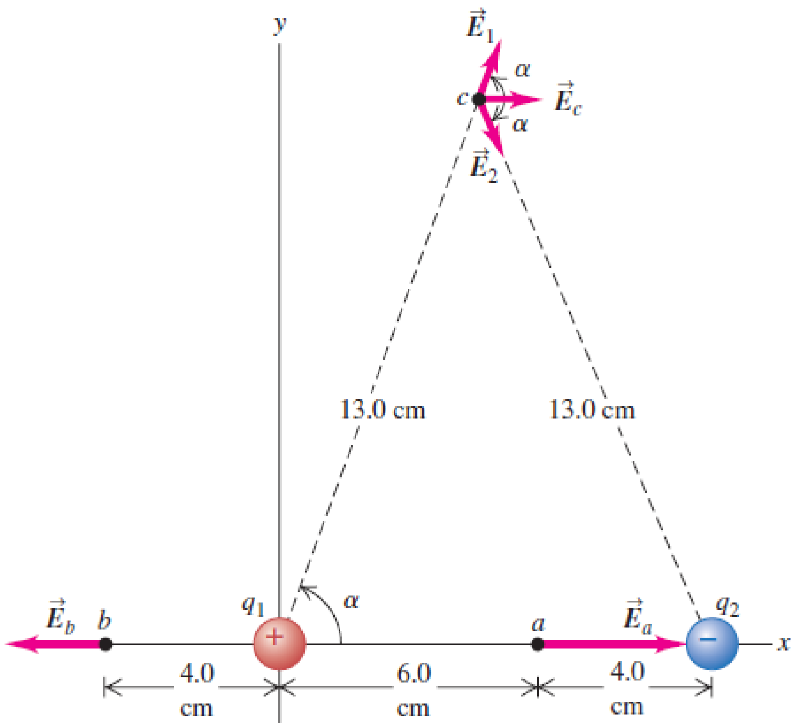
$$E_{1a} = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r^2} = (9.0 \times 10^9\ N \cdot m^2/C^2)\frac{12 \times 10^{-9}\ C}{(0.060\ m)^2}$$
$$= 3.0 \times 10^4\ N/C$$

We calculate the other field magnitudes in a similar way. The results are

$$E_{1a} = 3.0 \times 10^4\ N/C \qquad E_{1b} = 6.8 \times 10^4\ N/C$$
$$E_{1c} = 6.39 \times 10^3\ N/C$$
$$E_{2a} = 6.8 \times 10^4\ N/C \qquad E_{2b} = 0.55 \times 10^4\ N/C$$
$$E_{2c} = E_{1c} = 6.39 \times 10^3\ N/C$$

The *directions* of the corresponding fields are in all cases *away* from the positive charge q_1 and *toward* the negative charge q_2.

21.22 Electric field at three points, a, b, and c, set up by charges q_1 and q_2, which form an electric dipole.



Electric Field Lines

Michael Faraday, who introduced the idea of electric fields in the 19th century, thought of the space around a charged body as filled with *lines of force.* Although we no longer attach much reality to these lines, now usually called **electric field lines,** they still provide a nice way to visualize patterns in electric fields. ***Electric field lines extend away from positive charge (where they originate) and toward negative charge (where they terminate).*** and the direction of the field line at a point tells you what direction the force experienced by a charge will be if the charge is placed at that point. If the charge is positive, it will experience a force in the same direction as the field; if it is negative the force will be opposite to the field. The fields from isolated, individual charges look like this, fig.1.10.

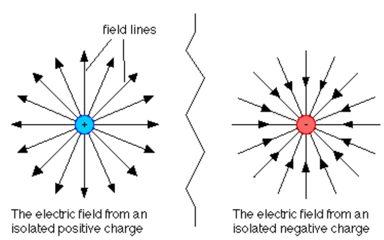


Figure 1.10

When there is more than one charge in a region, the electric field lines will not be straight lines; they will curve in response to the different charges. In every case, though, the field is highest where the field lines are close together, and decreases as the lines get further apart. A view of the lines of force depends upon arrangements of charges. The electric lines of force have got two properties:

1. They can be initiated and terminated at electric charges or at the infinity only;

2. They cannot cross with each other.

Examples of some electric fields are shown in the next figure 1.11.

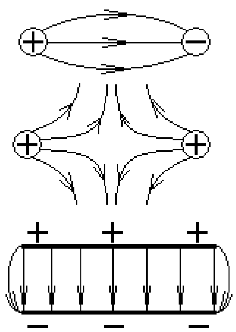


Figure 1.11

The Electric field around a charged conductor

A conductor is in the electrostatic equilibrium when the charge distribution (the way the charge is distributed over the conductor) is fixed. Basically, when you charge a conductor the charge spreads itself out. At equilibrium, the charge and electric field follow these guidelines:

- the excess charge lies only at the surface of the conductor;
- the electric field is zero inside of the conductor;
- the electric field at the surface of the conductor is perpendicular to the surface;
- charge accumulates, and the field is strongest, on pointy parts of the conductor.

Let's see if we can explain these things. Consider a negatively-charged conductor, in other words, a conductor with an excess of electrons. The excess electrons repel each other, so they want to get away from each other as far as possible. To do this they move to the conductor's surface. They also distribute themselves so the electric field inside the conductor is zero. If the field wasn't zero, any electrons that are free to move would. There are plenty of free electrons inside the conductor (they're the ones that are canceling out the positive charge from all the protons) and they don't move, so the field must be zero.

A similar argument explains why the field at the surface of the conductor is perpendicular to the surface. If it wasn't, there would be a component of the field along the surface. A charge experiencing that field would move along the surface in response to that field, which is inconsistent with the conductor being in equilibrium.

Why does the charge pile up at the pointy ends of the conductor? Consider two conductors, one in the shape of a circle and one in the shape of a line, fig 1.12. Charges are distributed uniformly along both conductors. With the circular shape, each charge has no net force on it, because there is the same amount of charge on either side of it and it is uniformly distributed. The circular conductor is in equilibrium, as far as its charge distribution is concerned.

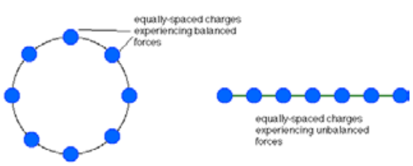


Figure 1.12

With the line, on the other hand, a uniform distribution does not correspond to equilibrium. If you look at the second charge from the left on the line, for example, there is just one charge to its left and several on the right. This charge would experience a force to the left, pushing it down towards the end. For charge distributed along a line, the equilibrium distribution would look more like this, fig. 1.13.

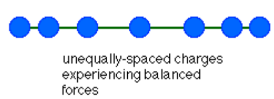


Figure 1.13

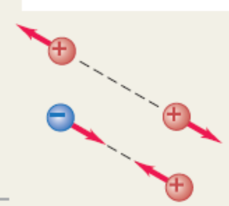
The charge accumulates at the pointy ends because that balances the forces on each charge.

Summary;

Electric charge, conductors, and insulators: The fundamental quantity in electrostatics is electric charge. There are two kinds of charge, positive and negative. Charges of the same sign repel each other; charges of opposite sign attract. Charge is conserved; the total charge in an isolated system is constant.

All ordinary matter is made of protons, neutrons, and electrons. The positive protons and electrically neutral neutrons in the nucleus of an atom are bound together by the nuclear force; the negative electrons surround the nucleus at distances much greater than the nuclear size. Electric interactions are chiefly responsible for the structure of atoms, molecules, and solids.

Conductors are materials in which charge moves easily; in insulators, charge does not move easily. Most metals are good conductors; most nonmetals are insulators.

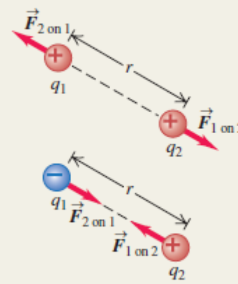


Coulomb's law: For charges q_1 and q_2 separated by a distance r, the magnitude of the electric force on either charge is proportional to the product q_1q_2 and inversely proportional to r^2. The force on each charge is along the line joining the two charges—repulsive if q_1 and q_2 have the same sign, attractive if they have opposite signs. In SI units the unit of electric charge is the coulomb, abbreviated C. (See Examples 21.1 and 21.2.)

When two or more charges each exert a force on a charge, the total force on that charge is the vector sum of the forces exerted by the individual charges. (See Examples 21.3 and 21.4.)

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2} \qquad (21.2)$$

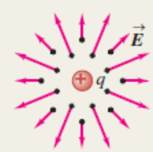
$$\frac{1}{4\pi\epsilon_0} = 8.988 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$$



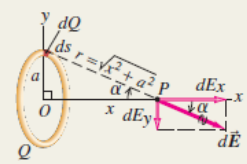
Electric field: Electric field $\vec{E}$, a vector quantity, is the force per unit charge exerted on a test charge at any point. The electric field produced by a point charge is directed radially away from or toward the charge. (See Examples 21.5–21.7.)

$$\vec{E} = \frac{\vec{F}_0}{q_0} \qquad (21.3)$$

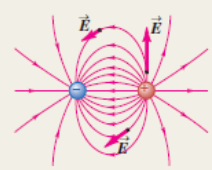
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \qquad (21.7)$$



Superposition of electric fields: The electric field $\vec{E}$ of any combination of charges is the vector sum of the fields caused by the individual charges. To calculate the electric field caused by a continuous distribution of charge, divide the distribution into small elements, calculate the field caused by each element, and then carry out the vector sum, usually by integrating. Charge distributions are described by linear charge density λ, surface charge density σ, and volume charge density ρ. (See Examples 21.8–21.12.)



Electric field lines: Field lines provide a graphical representation of electric fields. At any point on a field line, the tangent to the line is in the direction of $\vec{E}$ at that point. The number of lines per unit area (perpendicular to their direction) is proportional to the magnitude of $\vec{E}$ at the point.



••8 GO In Fig. 22-31, the four particles are fixed in place and have charges $q_1 = q_2 = +5e$, $q_3 = +3e$, and $q_4 = -12e$. Distance $d = 5.0\ \mu\text{m}$. What is the magnitude of the net electric field at point P due to the particles?

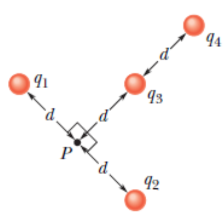


Fig. 22-31 Problem 8.

••9 GO Figure 22-32 shows two charged particles on an x axis: $-q = -3.20 \times 10^{-19}$ C at $x = -3.00$ m and $q = 3.20 \times 10^{-19}$ C at $x = +3.00$ m. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the net electric field produced at point P at $y = 4.00$ m?

••14 In Fig. 22-36, particle 1 of charge $q_1 = -5.00q$ and particle 2 of charge $q_2 = +2.00q$ are fixed to an x axis. (a) As a multiple of distance L, at what coordinate on the axis is the net electric field of the particles zero? (b) Sketch the net electric field lines.

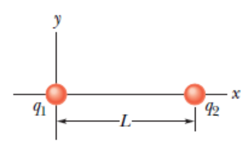


Fig. 22-36 Problem 14.

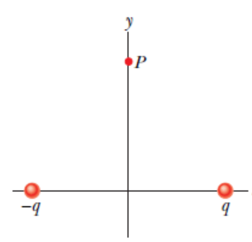


Fig. 22-32 Problem 9.